Received November 1997
Revised April 1999
Accepted May 1999

Improved solutions to a micropolar fluid driven by a continuous porous plate

André Desseaux
Laboratoire de Mécanique des Fluides et d'Energétique, Université de Valenciennes, Valenciennes, France, and
Mohammed Bellalij
Laboratoire d'Informatique et de Mathématiques Appliquées, Université de Valenciennes, Valenciennes, France

Keywords *Micropolar fluid, Quasilinearization*

Abstract *The theory of micropolar fluids was formulated by Eringen. A similarity solution is used to investigate the flow of such a fluid driven by a continuous porous plate. Continuous surfaces are surfaces such as polymer sheets or filaments continuously drawn from a dye. Within the framework of the boundary-layer theory, similarity transformation is used for the specific case when the wall velocity varies linearly with component. A physical characteristic of the fluid is used as a perturbation parameter to obtain a first estimate solution. Using a perturbation technique, analytical solutions for large transfer rates are presented. Then, a quasilinearization is used to obtain a complete solution. Good agreement is found between solutions obtained with these different methods and with the numerical data in Hassanien and Gorla (1990).*

Nomenclature

$\hat{u}; \hat{v}$	= velocity in $\hat{x}$ and $\hat{y}$-direction	ρ	= density of the fluid
$\hat{N}$	= angular velocity	$G; k.\rho.\nu$	= dimensionless material parameters
$\hat{x}; \hat{y}$	= distance along and normal to the surface	g	= dimensionless microrotation
$\eta = \sqrt{c/v}.\hat{y}$	= similarity variable	f	= dimensionless velocity function
ψ	= stream function	k	= material coefficient
$V = \hat{V}_0/\sqrt{c\nu}$	= dimensionless rate mass transfer		

Defining equations

In this paper, we study the flow of an incompressible micropolar fluid (see Eringen, 1996) driven by a stretching sheet subject to "suction" or "blowing". Hassanien and Gorla (1990) have numerically studied certain variations of this boundary value problem. Recently, one exact solution has been achieved both for the flow field and temperature distribution via a method of successive approximation (Hady, 1996). In all cases, a stipulated power law distribution for the temperature and the Eckert number depends on the longitudinal position. It should be pointed out that the relation between the exponents of velocity (m) and temperature (γ) distributions must be $\gamma = 2.m$ in order to obtain a real Eckert number (see Schlichting, 1968, chap. XII). Unfortunately, compared with a numerical solution, some (tiny) differences between these

International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 9 No. 7, 1999, pp. 730-741.

solutions are to be found particularly in the case of a negative exponent of the power law (see Hady, 1996, p. 102). The scope of this paper is to obtain various solutions for the flow field and to analyse the effects of the rate of mass transfer and of the characteristic k on the velocity and the wall friction coefficient.

According to Hassainien and Gorla (1990), the governing equations of a steady, incompressible micropolar fluid within the boundary layer approximation may be written as:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\hat{y}} = 0 \tag{1}$$

$$\hat{u}.\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}.\frac{\partial \hat{v}}{\partial \hat{y}} = \nu 1 \partial \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{k}{\rho} \cdot \frac{\partial \hat{N}}{\partial \hat{y}} \tag{2}$$

$$G.\frac{\partial^2 \hat{N}}{\partial \hat{y}^2} - 2.\hat{N} - \frac{\partial \hat{u}}{\partial \hat{y}} = 0 \tag{3}$$

In the above equations, $\hat{u}$ and $\hat{v}$ are the dimensional velocity components in the $\hat{x}$ and $\hat{y}$ directions, $\hat{N}$ represents the microrotation.

The conservation equations give, within the framework of the symplifying assumptions, a complete description of the physical occurences within the fluid. But, in order to complete the statement of the problem, we still have to specify the boundary conditions.

We assume that the velocity of a point on the porous plate is proportional to its distance from the leading edge ; it will be supposed that the no-slip condition of viscous flow continues to apply at the surface of the sheet. Furthermore, velocity $\hat{V}_0$ normal to the sheet specifies the mass injection or withdrawal:

$$\hat{y} = 0 : \hat{u} = c.\hat{x} = \hat{U}_w; \hat{v} = \hat{V}_0; \hat{N} = 0 \tag{4}$$

At large distances from the sheet, the conditions are as follows:

$$\hat{y} \to \infty; \quad \mathit{lim}\hat{u} = 0; \quad \mathit{lim}\hat{v} = 0; \quad \mathit{lim}\hat{N} = 0 \tag{5}$$

For a shape-preserving boundary-layer profile, we stipulate that there exists a stream function ψ (equation (1) is automatically satisfied) which depends on a classical similarity variable $\eta = \sqrt{c/\nu}.\hat{y}$. Using the definitions:

$$\psi = \sqrt{c.\nu.\hat{x}}.f(\eta); \hat{N} = \sqrt{c^3/\nu.\hat{x}}.g(\eta) \tag{6}$$

equations (2) and (3) are, respectively transformed into (primes denote the differentiation with respect to η):

$$f''' + f.f'' - f'^2 + \frac{k}{\rho.\nu}.g' = 0 \tag{7}$$

(there is a typographical error in equation (7) of Hassanien and Gorla (1990): term f has been omitted)

$$\frac{\mathrm{G.c}}{\nu} g'' = 2.g + f'' \tag{8}$$

In terms of the new variable, the transformed boundary conditions are:

$$\begin{gathered} f(0) = \frac{\hat{\mathrm{V}}_0}{\sqrt{c.\nu}} = V;\ f'(0) = 1;\ g(0) = 0 \\ \lim_{\eta\to\infty} f' = 0;\ \lim_{\eta\to\infty} g = 0 \end{gathered} \tag{9}$$

Physical property k: a perturbation development

According to Pipkin (1972), who wrote:

> In order to get correct answers when using slow motion approximations, it is necessary to use ordinary perturbation methods, starting from the Newtonian (Navier-Stokes) solution as the lowest-order approximation,

for small values of $\varepsilon = \mathrm{k}/\rho.\nu)$, we stipulate that the similarity functions may be expanded into the next regular perturbation expansions:

$$\begin{gathered} f = f_0 + \varepsilon.f_1 + \varepsilon^2.f_2 + \ldots \\ g = g_0 + \varepsilon.g_1 + \varepsilon^2.g_2 + \ldots \end{gathered} \tag{10}$$

and substitute them into the boundary-layer equations (7) and (8). Collecting terms in equal powers of ε, we can successively separate the different terms into third-order ordinary differential equations, the first four of which are:

$$\begin{gathered} f_0''' + f_0.f_0'' - f_0'^2 = 0 \\ f_\mathrm{k}''' + f_0.f_\mathrm{k}'' - 2.f_0'.f_\mathrm{k}' + f_\mathrm{k}.f_0'' = \begin{cases} -g_{\mathrm{k}-1}' & ;\mathrm{k}=1 \\ -g_{\mathrm{k}-1}' - f_1.f_1'' + f_1'^2 & ;\mathrm{k}=2 \\ -g_{\mathrm{k}-1}' - f_1.f_2'' - f_2.f_1'' + 2.f_1'.f_2' & ;\mathrm{k}=3 \end{cases} \end{gathered} \tag{11}$$

At any stage, the microrotation equation is identical to:

$$\frac{\mathrm{G.c}}{\nu} g_\mathrm{k}'' = 2.g_\mathrm{k} = f_\mathrm{k}''\ ; \mathrm{k} \geq 0 \tag{12}$$

Equations (11) and (12) are subjected to the following conditions:

$$\begin{gathered} f_0(0) = V\ , f_0'(0) = 1\ , g_0(0) = 0 \\ f_\mathrm{k}(0) = f_\mathrm{k}'(0) = g_\mathrm{k}(0) = 0\ ; \mathrm{k} \geq 1 \\ \lim_{\eta\to\infty} f_0' = 0\ ; \quad \lim_{\eta\to\infty} g_0 = 0 \\ \lim_{\eta\to\infty} f_\mathrm{k}' = 0\ ; \quad \lim_{\eta\to\infty} g_\mathrm{k} = 0\ ; \mathrm{k} \geq 1 \end{gathered} \tag{13}$$

At the leading order, the differential equation (11 ; k = 0) is nearly identical with a particular one obtained by Falkner and Skan. A previous study (Desseaux, 1998) gives an analytical solution:

$$
\begin{aligned}
f_0 &= V + (1 - \boldsymbol{B})/\beta = \beta - \boldsymbol{B}/\beta \\
\boldsymbol{B} &= exp(-\beta.\eta)\ ;\ \beta = \left(\sqrt{V^2+4} + V\right)/2
\end{aligned}
\tag{14}
$$

Introducing (14) in the first order equation (12) for g_0, we have:

$$
g_0'' - \frac{2.\nu}{\mathrm{G.c}}.g_0 = -\frac{\beta.\nu}{\mathrm{G.c}}.\boldsymbol{B} \tag{15}
$$

This ordinary differential equation is linear. Depending on the relative values of $2.\nu/(\mathrm{G.c})$ and β^2, the two alternative solutions are:

$$
\begin{aligned}
&\beta^2 \neq \frac{2.\nu}{\mathrm{G.c}} \Rightarrow g_0 = C[\boldsymbol{B} - \boldsymbol{R}] \\
&C = \frac{\beta}{2}\cdot\frac{r^2}{r^2 - \beta^2}\ ;\ r = +(2.\nu/(\mathrm{G.c}))^{1/2}\ ;\ \boldsymbol{R} = \exp(-r.\eta)
\end{aligned}
\tag{16}
$$

or

$$
\beta^2 \equiv \frac{2.\nu}{\mathrm{G.c}} \Rightarrow g_0 = \frac{\beta^2}{4}.\eta.\boldsymbol{B} \tag{17}
$$

For any value of $k = 1$, the homogeneous part of equation (11) can be written as:

$$
f_k''' + \beta.f_k''' - \frac{\boldsymbol{B}}{\beta}.\left(f_k'' + 2.\beta.f_k' + \beta^2.f_k\right) = 0 \tag{18}
$$

and we can immediately find a solution:

$$
\begin{aligned}
f_k &= \mathrm{A}_k.\boldsymbol{B} + \mathrm{B}_k.\left(\eta.\boldsymbol{B} + \beta - \frac{\boldsymbol{B}}{\beta}\right) \\
f_k &= \mathrm{A}_k.f_0' + \mathrm{B}_k.\left(\eta.f_0' + f_0\right)
\end{aligned}
\tag{19}
$$

Unfortunately, we have not found the third independent solution of (18). So, we need to use a numerical procedure to complete a solution.

Solutions for large mass transfer rates

In this section, we derive a solution of equations (7) and (8) valid for $|V|$ very large. We start considering the case of fluid withdrawal

Strong suction $\mathrm{V} \rightarrow \infty$

We start by putting

$$
\begin{aligned}
W &= |V|\ ;\ \delta = -\eta/W \\
f(\eta) &= -W.\varphi(\delta)\ ;\ g(\eta) = W.\psi(\delta)
\end{aligned}
\tag{20}
$$

Equations (7) and (8) become

$$\varphi'^2 - \varphi.\varphi'' = -\varphi'''/W^2$$
$$\psi = \frac{1}{2}\cdot\frac{1}{W^2}\cdot\left(\frac{\mathrm{G.c}}{\nu}.\psi'' + \varphi''\right) \tag{21}$$

subject to the following conditions:

$$\varphi(0) = 1\,;\varphi'(0) = 1\,;\quad \lim_{\delta\to\infty}\varphi' = 0$$
$$\psi(0) = 0\,;\quad \lim_{\delta\to\infty}\psi = 0 \tag{22}$$

Equations (21) suggest looking for a solution expanding

$$\varphi(\delta, W) = \varphi_0(\delta) + \frac{1}{W^2}.\varphi_1(\delta) + \ldots$$
$$\psi(\delta, W) = \psi_0(\delta) + \frac{1}{W^2}.\psi_1(\delta) + \ldots \tag{23}$$

At leading order, we obtain a solution satisfying the conditions (22):

$$\varphi_0 = exp(\delta) = \boldsymbol{D};\quad \psi_0 = 0 \tag{24}$$

The continuation to $O(W^{-2})$ is straightforward. The first resulting equation is:

$$\varphi_1'' - 2.\varphi_1'' + \varphi_1 = -1 \tag{25}$$

The solution is:

$$\varphi_1 = \boldsymbol{D} - \delta.\boldsymbol{D} - 1 \tag{26}$$

Introducing (23a) in (8), equation for g needs to be:

$$\frac{\mathrm{G.c}}{\nu}.g'' - 2.g = \frac{\boldsymbol{D}}{W}\cdot\left(\frac{1+\delta}{W^2} - 1\right) \tag{27}$$

The solution for this equation is:

$$g = K_1\boldsymbol{D} + K_2.\eta.\boldsymbol{D} + K_3.\boldsymbol{R} \tag{28}$$

with

$$K_1 = \frac{1}{W}\cdot\frac{1}{2.W^2 - \mathrm{G.c}/\nu}\cdot\left[W^2 - 1 - \frac{2.\mathrm{G.c}/\nu}{2.W^2 - \mathrm{G.c}/\nu}\right]$$
$$K_2 = \frac{1}{W^2}\cdot\frac{1}{2.W^2 - \mathrm{G.c}/\nu}\,;K_3 = -K_1 \tag{29}$$

The dimensionless wall shear stress $f''(0)$ and the gradient of microrotation $g'(0)$ are calculated and lead to:

$$f''(0) \approx W^{-1}.\left(W^{-2} - 1\right)$$
$$g'(0) \approx K_1.\left(r - W^{-1}\right) + K_2\,;\, r = +(2.\nu/(\mathrm{G.c}))^{1/2} \tag{30}$$

On Figures 1 and 2, we plot values of $-f''(0)$ and $g'(0)$ obtained from numerical integration of equations (7) and (8) for two values of $\mathrm{k}/\rho.\nu$. We can see that these curves approach the asymptotic forms (30a) and (30b) as $W \to \infty$. For instance, the variation of $\mathrm{k}/\rho.\nu$ is negligible as soon as $W > 4.0$.

Strong injection $\mathrm{V} \to \infty$

The solution of equation (7) is relatively standard and in keeping with a previous study (Desseaux, 1998), we put:

$$f(\eta) = V + \frac{\Phi(\chi)}{V}\,; g(\eta) = \frac{\Psi(\chi)}{V}\,; \chi = V.\eta \tag{31}$$

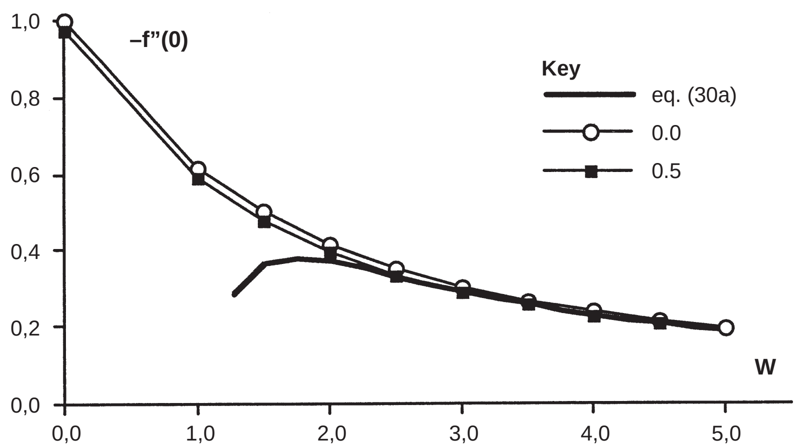

Figure 1.
Variation of the dimensionless wall shear stress $-f''(0)$ equation (30a); influence of the physical parameter $\mathrm{k}/\rho.\nu$ using the quasilinearization scheme ($\mathrm{k}/\rho.\nu = 0$ = dots; $\mathrm{k}/\rho.\nu = 0.5$ = squares); $\mathrm{G.c}/\nu = 2.0$

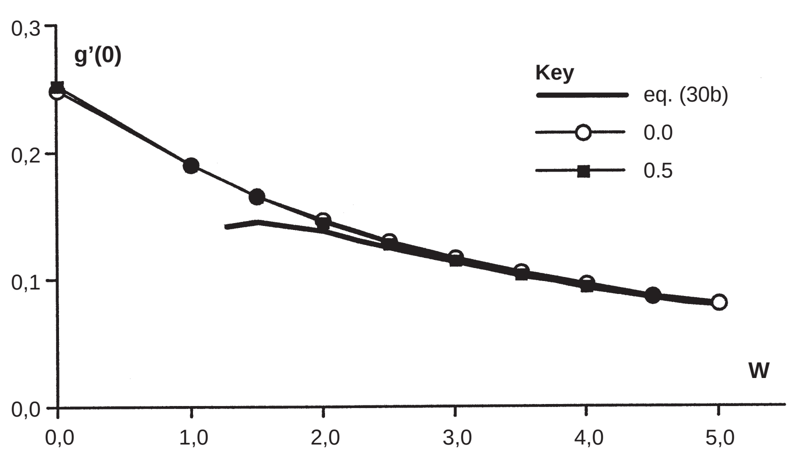

Figure 2.
Variation of the wall gradient of microrotation $g(0)$ equation (30b); influence of the physical parameter $\mathrm{k}/\rho.\nu$ using the quasilineatization scheme ($\mathrm{k}/\rho.\nu = 0$ = dots; $\mathrm{k}/\rho.\nu = 0.5$ = squares); $\mathrm{G.c}/\nu = 2.0$

Equations (7) and (8) become

$$\Phi''' + \Phi'' = \left(\Phi'^2 - \Phi.\Phi'' - \frac{\mathrm{k}}{\rho.\nu}\Psi'\right)V^2$$
$$\Psi'' - \Phi''/\frac{\mathrm{G.c}}{\nu} = \left(2.\Psi/\frac{\mathrm{G.c}}{\nu}\right)/V^2 \tag{32}$$

subject to the following conditions:

$$\Phi(0) = 0\,;\Phi'(0) = 1\,;\quad \lim_{\chi\to\infty}\Phi' = 0$$
$$\Psi(0) = 0\,;\quad \lim_{\chi\to\infty}\Psi = 0 \tag{33}$$

Equations (32) suggest looking for a solution expanding

$$\Phi(\chi, V) = \Phi_0(\chi) + \frac{1}{V^2}.\Phi_1(\chi) + \ldots$$
$$\Psi(\chi, V) = \Psi_0(\chi) + \frac{1}{V^2}.\Psi_1(\chi) + \ldots \tag{34}$$

At leading order, we obtain a solution satisfying conditions (33a):

$$\Phi_0(\chi) = 1 - exp(-\chi) = 1 - \boldsymbol{X} \tag{35}$$

but to obtain a solution for the microrotation verifying all the conditions (33b), we need to use the next equation:

$$\frac{\mathrm{G.c}}{\nu}.\Psi'' - \frac{2}{V^2}.\Psi = \Phi_0'' = -\boldsymbol{X} \tag{36}$$

The solution for this equation is:

$$\Psi_0 = \frac{\nu}{\mathrm{G.c}}.\frac{1}{\sigma^2 - 1}.(\boldsymbol{X} - \boldsymbol{R})\,;\boldsymbol{R} = exp(-r.\eta)\,;r = \sqrt{\frac{2.\nu}{\mathrm{G.c}}}\,;\sigma = \frac{r}{V} \tag{37}$$

The continuation to $O(W^{-2})$ is straightforward. The first resulting equation is:

$$\Phi_1''' + \Phi_1'' = \left(1 + \frac{\mathrm{k}}{\rho.\nu}.\frac{\nu}{\mathrm{G.c}}.\frac{1}{\sigma^2 - 1}\right).\boldsymbol{X} - \left(\frac{\mathrm{k}}{\rho.\nu}.\frac{\nu}{\mathrm{G.c}}.\frac{\sigma}{\sigma^2 - 1}\right).\boldsymbol{R} \tag{38}$$

The solution is:

$$\Phi_1 = Q_1.\chi.\boldsymbol{X} + Q_2.\boldsymbol{X} + Q_3.\boldsymbol{R} + Q_4 \tag{39}$$

with

$$Q_1 = 1 + \frac{\mathrm{k}}{\rho.\nu}\cdot\frac{\nu}{\mathrm{G.c}}\cdot\frac{1}{\sigma^2-1}\,;Q_3 = \frac{Q_1-1}{\sigma.(\sigma-1)}$$
$$Q_2 = \frac{Q_1.(\sigma-2)+1}{\sigma-1}\,;Q_4 = \frac{Q_1.(1-\sigma)-1}{\sigma} \tag{40}$$

At this stage, it is interesting to observe that:

- the solution depends only on the ratio of the two physical parameters k/G;
- the dimensionless coefficient of the wall shear stress is independent of the physical parameters $\mathrm{k}/\rho.\nu$ and $\mathrm{G.c}/\nu$.

The calculation of the unknown wall derivatives gives:

$$f''(0) \approx -\left[V + V^{-1}\right];g'(0) \approx \frac{\nu}{\mathrm{G.c}}\cdot\frac{1}{\sigma+1} \tag{41}$$

We solved equations (7) and (8) numerically for increasing values of V for two values of the physical parameter $\mathrm{k}/\rho.\nu$. The results for $-f''(0)$ and $g'(0)$ are shown in Figures 3 and 4. Asymptotic expression (41) is also shown in these figures. We can see that the solutions are in very good agreement even at quite moderate values of V; the curves are approaching each other as required by the theory. The validity of development (34a) limited to the second order is acceptable if the wall mass transfer is greater than 3.0. We need to calculate the second order approximation for (34b) with results (35) and (39).

Numerical procedures

Solution of (11) and (12)

In this section, we consider the different approximations $f_{\mathrm{k}}(\mathrm{k} \geq 1)$ of equations (11) which are linear. So, we may construct a solution $S_{\mathrm{k}} = [f, f', f'', g, g']^t$ with a linear combination (depending on a parameter ω) of two independent solutions which are:

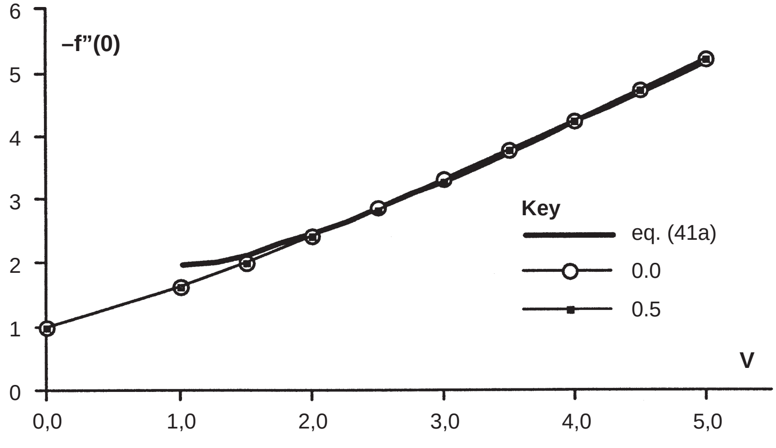

Figure 3. Comparison of the dimensionless wall shear stress $-f''(0)$; influence of the physical parameter $\mathrm{k}/\rho.\nu$ equation (41a) with quasilinearization (dots $= \mathrm{k}/\rho.\nu = 0$; squares $= \mathrm{k}/\rho.\nu = 0.5$) $\mathrm{G.c/nu} = 2.0$

(1) A particular solution Sp of equations (11) and (12) ($k \geq 1$) with conditions

$$S_p(0) = \left[o, o, f''_{k-1}(0), 0, g'_{k-1}(0)\right]^t \quad (42)$$

(for the first step, we have used $S''_1(0) = 0$).

(2) A solution for the homogeneous part

$$f'''_k + f_0.f''_k - 2.f'_0.f'_k + f_k.f''_0 = 0$$
$$\frac{G.c}{\nu}.g''_k - 2.g_k = 0 \quad (43)$$

with initial conditions $S_h(0) = [0, 0, 1, 0, 1]^t$.

Parameter ω in the combination $S = S_p + \omega.S_h$ can be split up into two coefficients. These coefficients are calculated, in a single step, with equality (37) deduced from condition at infinity (15):

$$0 = f'_p(\eta_{max}) + \omega_f.f'_h(\eta_{max})\,; 0 = g_p(\eta_{max}) + \omega_g.g_h(\eta_{max}) \quad (44)$$

Here η_{max} is "sufficiently large" to ensure that $exp(-\beta.\eta_{max}), f'_0(\eta_{max})$, and $g_0(\eta_{max})$ are less than a typical value (e.g. 10^{-4}). For instance, in case of moderate suction ($V \leq -3.0$) equation (14) gives $\beta \approx 0.30$ and we have found $f'(30) > 10^{-4}$.

Quasilinearization method

Equation (7) represents a two-point non-linear third-order differential equation. To compare the solution obtained with developments (10), (20) and (31) and check the validity, we have solved the set of equations (7) and (8) using a quasi-linearization scheme described in Desseaux (1998) and Zagustin *et al.* (1978).

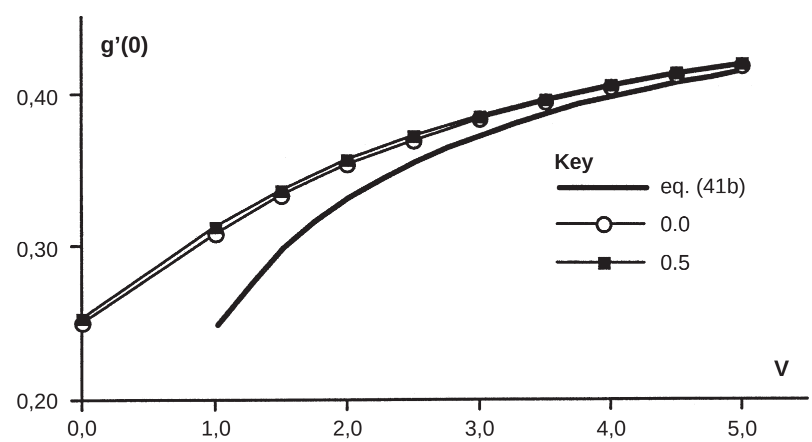

Figure 4. Variation of the wall gradient of micrrotation $g(0)$equation (41b) (plain line); influence of the physical parameter $k/\rho.\nu$ using the quasilinearization scheme ($k/\rho.\nu = 0$ = dots; $k/\rho.\nu = 0.5$= squares); $G.c/\nu = 2.0$

Let $f_{(j)}, g_{(j)}$ be an approximate solution and $f_{(j+1)}, g_{(j+1)}$, an improved solution of system (7)-(8). A first order Newton's development around the former solution gives the two linear coupled equations

$$
\begin{aligned}
f'''_{(j+1)} &= -f''_{(j+1)}.f_{(j)} + 2.f'_{(j+1)}.f'_{(j)} - f_{(j+1)}f''_{(j)} - \frac{k}{\rho.\nu}g'_{(j+1)} \\
&\quad + \left(f''_{(j)}.f_{(j)} - f'^2_{(j)}\right) \\
g''_{(j+1)} &= \frac{\nu}{G.c}\left[2.g_{(j+1)} + f''_{(j+1)}\right]
\end{aligned} \tag{45}
$$

The numerical procedure used also a linear combination $S = S_p + \zeta.S_{h1} + \xi.S_{h2}$ where S_p and S_h represent respectively a particular solution of system (45) and solutions of the homogeneous part which verify the following initial conditions:

$$
\begin{cases}
S_p(0)|_{(j+1)} = \left[V, 1, f''_{(j)}(0), 0, g'_{(j)}(0)\right]^t \\
S_{h1}(0) = [0, 0, 1, 0, 0]^t \\
S_{h2}(0) = [0, 0, 0, 0, 1]^t
\end{cases} \tag{46}
$$

The first estimate values are given by $f_{(0)} = f_0$ (14) and $g_{(0)} = g_0$ (16) or (17). Verifying the conditions at infinity (9b), the weighting coefficients ζ and ξ are calculated at each stage from the two next equations:

$$
0 = f'_p + \zeta.f'_{h1} + \xi.f'_{h2}|_{\eta=\eta_{\max}}; 0 = g'_p + \zeta.g_{h1} + \xi.g_{h2}|_{\eta=\eta_{\max}} \tag{47}
$$

Results

Remarks

A typographical mistake (one sign) is to be found in Hady's analytical solution. Equation (22) of Hady (1996) should be read as:

$$
\begin{aligned}
f &= V + A - B - A.exp(-\beta\eta) - D.\left(\frac{1}{\beta}.exp(-\beta\eta) - \frac{\beta.}{r^2}.exp(-r\eta)\right) \\
A &= \frac{V.\beta+1}{\beta^3}; B = \frac{k}{2.\rho.\nu}.\frac{1}{\beta}; D = \frac{k}{2.\rho.\nu}.\frac{r^2}{\beta^2 - r^2}; r = \sqrt{\frac{2.\nu}{G.c}}
\end{aligned} \tag{48}
$$

Comparison

Table I indicates the effect of the mass transfer parameter V on the wall friction. We observe that suction $V < 0$ reduces the friction factor as well as the wall gradient of microrotation, whereas the injection $V > 0$ has the opposite effect. Our numerical values agree well with Hassanien and Gorla (1990); our "approximate" solutions obtained with a perturbation technique (10) are closer to the values of Hassanien and Gorla (1990) than those of Hady (1996).

In Figure 5 we have shown the microrotation distribution for a moderate suction. We observe that our development (28) is in very good agreement with our numerical results obtained with a quasilinearization scheme. A difference is marked only when the physical parameter $k/\rho.\nu$ is greater than 0.5.

In Figure 6 we analyse the relative influence between the two physical parameters $k/\rho.\nu$ and $G.c/\nu$ using the quasilinearization scheme. For the plain line, we have used $k/\rho.\nu$ = 0.2 and $G.c/\nu$ = 1.0. For the other line, these parameters are equal to 1.0 and 5.0. We can see that the results are quite identical for the same ratio $G.\rho.c/k$. We compare these results

Table I. Comparison of the missing wall gradient values $-f''(0)$ and $g'(0)$. Values issued from Hady (1996) and Hassanien and Gorla (1990), our perturbation development (11) and (12) and a quasilinearization scheme (45); $Gc/\nu = 2.0$ $k/\rho\nu = 0.2$

	Hady (1996) equation (48)	Hassanien and Gorla (1990)	(11)	(45a)
$-f''(0)$				
V				
–0.7		0.69979	0.6984	0.69961
0	0.97500	0.99081	0.99075	0.99098
0.4	1.19781	1.21183	1.2162	1.2118
0.7	1.39003	1.40265	1.4136	1.4027
	Hady (1996)	Hassanien and Gorla (1990)	(12)	(45b)
$g'(0)$				
V				
–0.7		0.20819	0.20718	0.20797
0	0.25316	0.25121	0.25119	0.25129
0.4	0.27812	0.27622	0.27761	0.27622
0.7	0.29585	0.29405	0.29696	0.29408

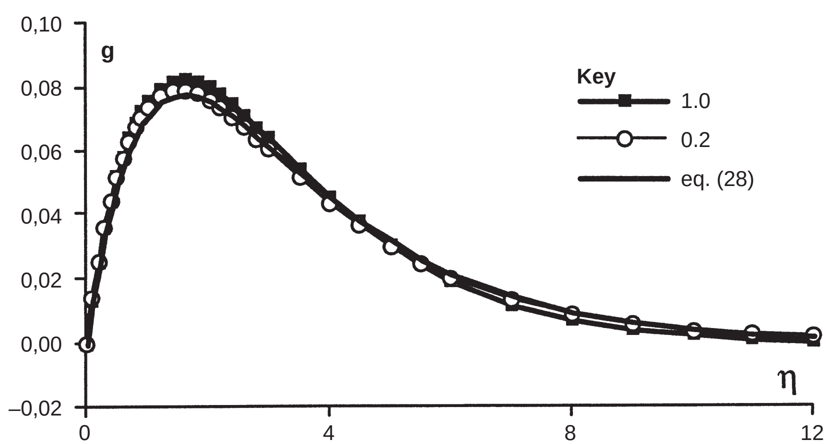

Figure 5. Comparison of the profiles of microrotation $g(\eta)$; influence of the physical parameter $k/\rho.\nu$ equation (28) (dots $k/\rho.\nu$ = 0.2; squares $k/\rho.\nu$ = 1.0); $G.c/\nu$ = 2.0; $W = |V|$ = 2.0

with the dimensionless velocity profiles issued from the asymptotic development (34a). It gives quite identical results for the same values of $G.c/\nu$. We observe that for the two cases ($G.c/\nu = 1.0$ dots ; $G.c/\nu = 5.0$ black squares) decreasing the parameter $k/\rho.\nu$ decreases the boundary-layer thickness.

Conclusion

In this paper, we have used the theory of micropolar fluids formulated by Eringen (1966), to derive a set of boundary-layer equations for the micropolar fluid flow over a stretching sheet. We have considered in some detail the similarity solutions considering the influence of the physical parameters. A perturbation technique has been used to analyse the influence of microrotation and wall mass transfer on the velocity distribution. A numerical procedure using the quasilinearization scheme has been used to compare all the previously published data and to confirm our developments.

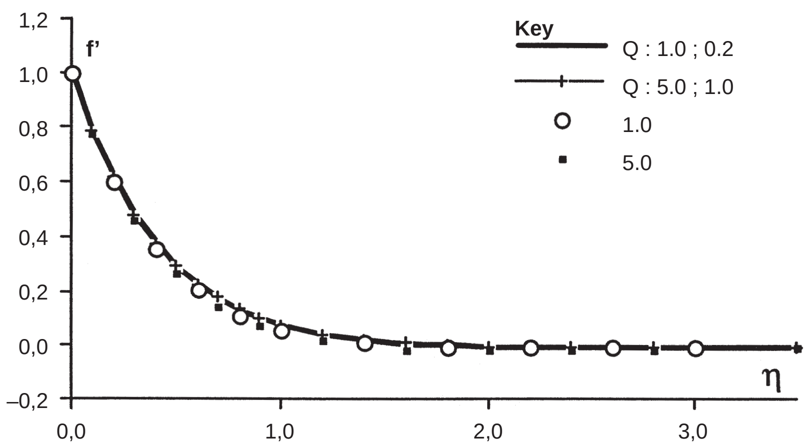

Figure 6. Velocity profiles $f'(\eta)$ (Q = quasilinearization); mutual influence of the physical parameters $k/\rho.\nu$ (0.2 or 1.0) and $G.c/\nu$ (1.0 or 5.0) – comparison with the asymptotic development (34) (dots $G.c/\nu = 1.0$ or black squares = 5.0) – $V = 2.0$

References

Desseaux, A. (1998), "Analytical and numerical solutions to a problem of convection in a porous media with lateral mass flux", *Int. Comm. in Heat & Mass Trans.*, Vol. 5 No. 5, pp. 641-50.

Eringen, A.C. (1966), "Theory of micropolar fluids", *J. Math. Mech.*, Vol. 16 No. 1.

Hady, F.M. (1996), "On the solution of heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection", *Int. J. Num. Heat Fluid Flow*, Vol. 6, pp. 99-104.

Hassanien, I.A. and Gorla, R.S.R. (1990), "Heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing", *Acta Mech.*, Vol. 84, pp. 191-9.

Pipkin, A.C. (1972), *Lectures on Viscoelasticity Theory*, Springer Verlag, Berlin.

Schlichting, H. (1968), *Boundary-Layer Theory*, 6th ed., McGraw-Hill, New York, NY.

Zagustin, E., Kalaba, R. and Ikeda, G. (1978), "Solution of the Falkner-Skan equation by quasi-linearization – numerical methods in laminar and turbulent flow", *Proceedings of the First Int. Conf.*, Pentech Press, Swansea.